Prove the quotient rule using the definition of the derivative function. **Show all steps.** SCORE: _____ / 15 PTS

$$\left(\frac{f(x)}{g(x)}\right)' = \lim_{h \to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

2

$$= \lim_{h \to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}$$

2

$$= \lim_{h \to 0} \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}$$

3

$$= \frac{\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h} \lim\limits_{h \to 0} g(x) + \lim\limits_{h \to 0} f(x) \lim\limits_{h \to 0} \frac{g(x)-g(x+h)}{h}}{\lim\limits_{h \to 0} g(x+h)g(x)}$$

$2\frac{1}{2}$ $\quad$ $2\frac{1}{2}$ $\quad$ 2

$$= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

1

If $f(x) = \tan^{-1} x^3$, find $f''(-1)$.

SCORE: _____ / 20 PTS

$$f'(x) = \frac{1}{1+(x^3)^2} \cdot 3x^2 = \frac{\boxed{3x^2}\ 4}{\boxed{1+x^6}\ 4}$$

$$f''(x) \overset{3}{=} \frac{\boxed{6x(1+x^6)} - \boxed{3x^2(6x^5)}\ 3}{\boxed{(1+x^6)^2}\ 3}$$

$$f''(-1) = \frac{-6(2)-3(-6)}{2^2} = \boxed{\frac{3}{2}}\ 3$$

Find $\frac{d}{dx}(\csc x)^{\tan x}$. **SCORE: _____ / 15 PTS**

$$y=(\csc x)^{\tan x}$$

$$3\ \boxed{\ln y=\tan x\ln\csc x}$$

$$4$$

$$\boxed{\frac{1}{y}\frac{dy}{dx}}=\boxed{\sec^2 x\ln\csc x}+\boxed{\cancel{\tan} x\frac{1}{\cancel{\csc x}}(-\cancel{\csc}x\cancel{\cot}x)}=\sec^2 x\ln\csc x-1$$

$$2 \qquad 2$$

$$\frac{dy}{dx}=y(\sec^2 x\ln\csc x-1)=\boxed{(\csc x)^{\tan x}}\boxed{(\sec^2 x\ln\csc x-1)}$$

$$2 \qquad 2$$

The following table gives values and derivatives of a function at various inputs. **SCORE: _____ / 15 PTS**

x	-3	-2	-1	0	1	2	3	4
$f(x)$	-2	0	2	4	-3	-1	1	3
$f'(x)$	4	-1	-3	2	-4	3	-2	1

If $m(x) = 2^{f(x)}$, find the equation of the tangent line to $y = m(x)$ at $x = -3$.

$m(-3) = 2^{f(-3)} = 2^{-2} = \boxed{\frac{1}{4}}$ 3

$m'(x) = \boxed{2^{f(x)}(\ln 2) f'(x)}$ 6

$m'(-3) = 2^{f(-3)}(\ln 2) f'(-3) = 2^{-2}(\ln 2)4 = \boxed{\ln 2}$ 3

$\boxed{y - \frac{1}{4} = (\ln 2)(x+3)}$ 3

Let $f(x) = \cos^{-1} \frac{2}{x}$.

SCORE: _____ / 20 PTS

[a] If x changes from 4 to 3.8, find dy.

$$f'(x) = \boxed{-\frac{1}{\sqrt{1-\left(\frac{2}{x}\right)^2}}} \cdot \boxed{\frac{-2}{x^2}} \quad 4 \quad 4$$

$$f'(4) = -\frac{1}{\sqrt{1-\frac{1}{4}}} \cdot \frac{-2}{16}$$

$$= \frac{2}{\sqrt{3}} \cdot \frac{1}{8} = \boxed{\frac{1}{4\sqrt{3}}} \quad 2$$

$$dy = \frac{1}{4\sqrt{3}}(3.8-4)$$

$$= \frac{1}{4\sqrt{3}} \cdot \boxed{\frac{-1}{5}} \quad 3$$

$$= \boxed{\frac{-1}{20\sqrt{3}}} \quad 2$$

[b] Approximate $f(3.8)$ **using your answer to part [a]**.

$$f(3.8) \approx f(4) + dy = \cos^{-1}\frac{2}{4} - \frac{1}{20\sqrt{3}} = \boxed{\frac{\pi}{3}} \boxed{-\frac{1}{20\sqrt{3}}} \quad 2\tfrac{1}{2} \quad 2\tfrac{1}{2}$$

The position of an object at time t minutes is given by the function $s(t) = \dfrac{t^3 - 9t + 15}{10\sqrt[3]{t}}$ yards for $t \ge 0.5$.

SCORE: _____ / 15 PTS

Find the acceleration of the object at time $t = 1$ minute.

$$s(t) = \frac{1}{10}t^{\frac{8}{3}} - \frac{9}{10}t^{\frac{2}{3}} + \frac{3}{2}t^{-\frac{1}{3}}$$

$$s'(t) = \boxed{\frac{4}{15}t^{\frac{5}{3}} - \frac{3}{5}t^{-\frac{1}{3}} - \frac{1}{2}t^{-\frac{4}{3}}} \quad 5$$

$$s''(t) = \boxed{\frac{4}{9}t^{\frac{2}{3}} + \frac{1}{5}t^{-\frac{4}{3}} + \frac{2}{3}t^{-\frac{7}{3}}} \quad 5$$

$$s''(1) = \frac{4}{9} + \frac{1}{5} + \frac{2}{3} = \frac{20 + 9 + 30}{45} = \boxed{\frac{59}{45}} \; \boxed{\text{YARDS/MINUTE}^2} \quad 2$$

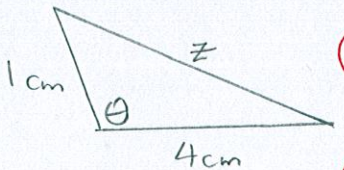
3

Two sides of a triangle have lengths 1 cm and 4 cm. At the moment when the angle between those two sides is 120°, that angle is shrinking by 10° per minute. How quickly is length of the third side changing at that moment ? **SCORE: _____ / 25 PTS**

You must state whether the third side is getting longer or shorter.

You must state/show clearly what each variable you use represents.

You must show the units during the intermediate steps of your work, and you must state the units for the final answer.



$$\left.\frac{d\theta}{dt}\right|_{\theta=\frac{2\pi}{3}} = \frac{-\pi}{18}/\text{MINUTE}$$

WANT $\left.\dfrac{dz}{dt}\right|_{\theta=\frac{2\pi}{3}}$

@ $\theta = \frac{2\pi}{3}$

$$z^2 = \left(17 - 8\cos\tfrac{2\pi}{3}\right)\text{cm}^2 = 21\,\text{cm}^2$$

$$z = \sqrt{21}\ \text{cm}$$

$5\frac{1}{2}$ $\boxed{z^2 = (1\,\text{cm})^2 + (4\,\text{cm})^2 - 2(1\,\text{cm})(4\,\text{cm})\cos\theta = (17 - 8\cos\theta)\,\text{cm}^2}$ 1

$5\frac{1}{2}$ $\boxed{\cancel{2}z\frac{dz}{dt} = \cancel{8}^{\,4}(\sin\theta)\frac{d\theta}{dt}\,\text{cm}^2}$

2

$$\boxed{\sqrt{21}\ \cancel{\text{cm}}}\ \frac{dz}{dt} = 4\left(\sin\frac{2\pi}{3}\right)\boxed{\left(\frac{-\pi}{18}/\text{MINUTE}\right)}\ \text{cm}^{\cancel{2}}$$

3

$$\frac{dz}{dt} = -\cancel{4}\boxed{\left(\frac{\sqrt{3}}{\cancel{2}}\right)}\left(\frac{\pi}{\cancel{18}_{\,9}}\right)\left(\frac{1}{\cancel{\sqrt{21}}_{\,\sqrt{7}}}\right)\frac{\text{cm}}{\text{MINUTE}}$$ 1

3 $$= \boxed{\frac{-\pi}{9\sqrt{7}}}\ \frac{\text{cm}}{\text{MINUTE}}$$

THE THIRD SIDE IS $\boxed{\text{GETTING SHORTER}}$ 2

BY $\boxed{\dfrac{\pi}{9\sqrt{7}}}$ $\boxed{\text{cm PER MINUTE}}$ 1

1

If $\cos y^2 - \sin^2 x = x^3 e^{2y}$, find $\frac{dy}{dx}$. **SCORE: _____ / 25 PTS**

6 4 3 6

$$\boxed{(-\sin y^2)\, 2y \frac{dy}{dx}} - \boxed{2\sin x \cos x} = \boxed{3x^2 e^{2y}} + \boxed{x^3 \left(2e^{2y} \frac{dy}{dx}\right)}$$

$$-2\sin x \cos x - 3x^2 e^{2y} = (2x^3 e^{2y} + 2y \sin y^2) \frac{dy}{dx}$$

$$\boxed{-\frac{2\sin x \cos x + 3x^2 e^{2y}}{2x^3 e^{2y} + 2y \sin y^2}} = \frac{dy}{dx}$$

6